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AUTHOR O'Brien, Peter; Richardson, Arthur
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ABSTRACT

This report: (1) describes the mathematics test given to Year 9 students in the Mt. Druitt Longitudinal Study in 1987; (2) describes and discusses the results; (3) analyzes errors made by students; (4) analyzes the responses to items common to the 1986 and 1987 tests; and (5) suggests some interventions to address the errors made by students. The study dates from 1975 when funds were obtained to provide intervention programs at five preschools in Mt. Druitt (Australia), an area which had been identified as being disadvantaged. These intervention programs were aimed at bridging the educational achievement gap between these "disadvantaged" students and those who were not "disadvantaged." The intervention programs ceased in 1979 when the children had completed first grade. Since that time the study has been involved in assessing educational skills, pupil attitudes, and teacher and pupil classroom behavior through testing, questionnaires, and observation. Teacher and parent interviews have also been conducted. To determine whether the early intervention assisted the Mt. Druitt students, comparison schools were chosen to undergo the same testing as the Mt. Druitt schools. In 1987, the students who were identified in 1977 were in year 9 in high school. Scripts of the 1987 test, which was administered in two parts on separate occasions within one school week in April, are appended. (PK)

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MT. DRUITT LONGITUDINAL STUDY

1987 MATHEMATICS INVESTIGATION

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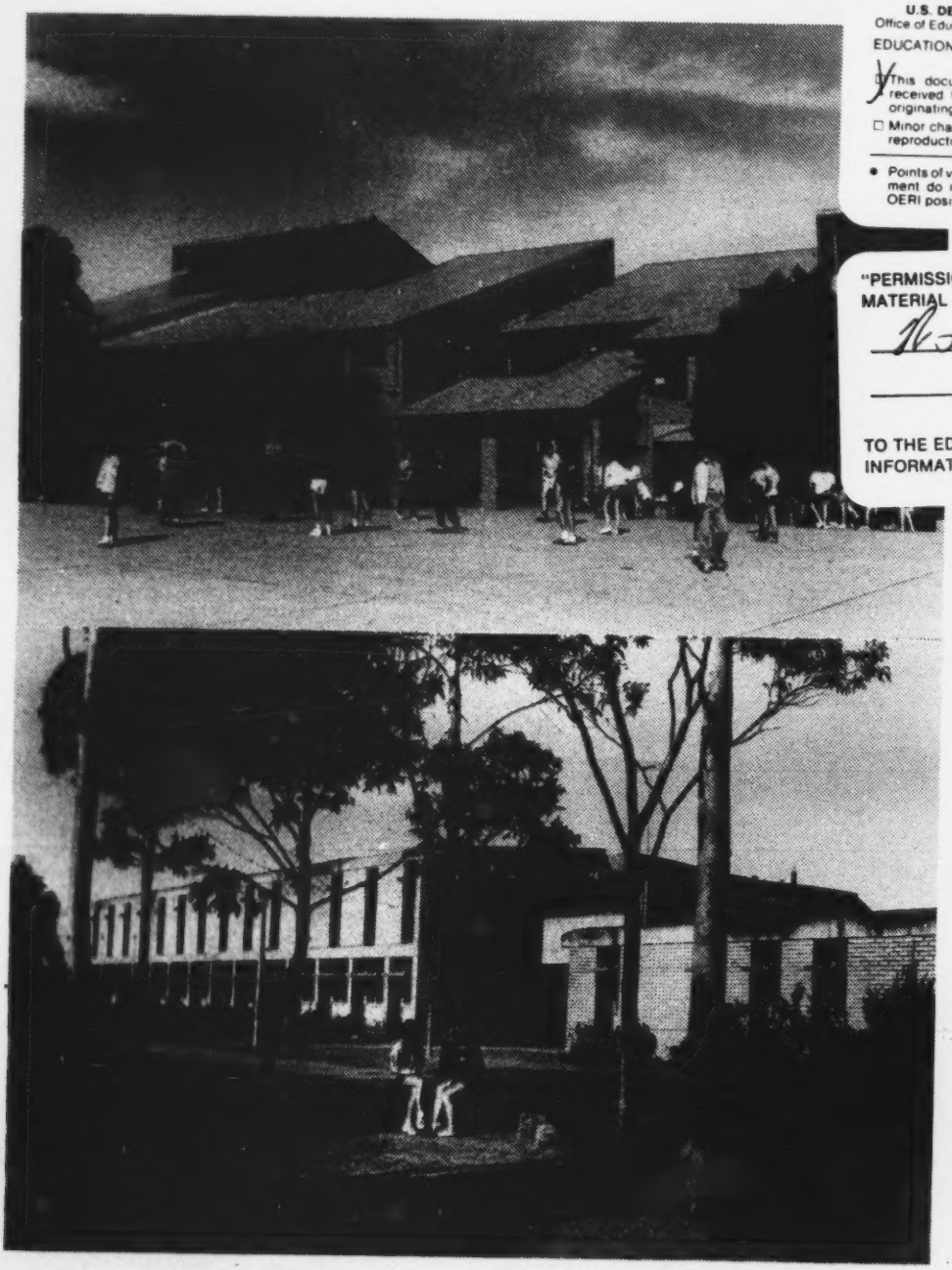

Peter O'Brien and Arthur Richardson

SCHOOL OF EDUCATION • MACQUARIE UNIVERSITY, AUSTRALIA 2109

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MT DRUITT LONGITUDINAL STUDY

1987 MATHEMATICS INVESTIGATION

A Report on results,
including an analysis of errors and
suggestions for remediation

Peter O'Brien and Arthur Richardson

July, 1987

Acknowledgements

The Principals of the schools in the Study are gratefully acknowledged for their cooperation, as are the mathematics teachers for their assistance in administering the test.

BACKGROUND TO THE MT. DRUITT LONGITUDINAL STUDY

The Study dates from 1975 when funds were obtained from the Bernard van Leer Foundation to provide intervention programs at five preschools in Mt. Druitt, west of Sydney, an area which had been identified, by welfare agencies, as being disadvantaged. All Mt. Druitt schools were at that time identified as disadvantaged. These intervention programs were aimed at bridging the educational achievement gap between these "disadvantaged" students and those who were not "disadvantaged". The intervention programs ceased in 1979 when the children had completed first grade. Since that time the Study has been involved in assessing educational skills, pupil attitudes and teacher and pupil classroom behaviour through testing, questionnaires and observation. As well teacher and parent interviews have been conducted.

To determine whether the early intervention assisted the Mt. Druitt students and how it was that "disadvantage" operated, comparison schools were chosen to undergo the same testing as the Mt. Druitt schools. At present, 1987, the students who were first identified in preschool in 1977, are in Year 9 at High School and the Study has students in three high schools in the Mt. Druitt area, and in two non-disadvantaged contrast high schools, one of which is near Sydney whilst the other is distant from Sydney. The number of students from these two contrast high schools who were in the study in primary school has reduced greatly, due largely to many of them going to private schools. There are significant problems associated with longitudinal studies, one is the loss of students from the study through reasons such as changing schools and moving to a different area. To counter such losses, extra students were included in the sample in 1986 and a new, high school based longitudinal study commenced. It is planned that this study should continue for at least two years.

TECHNICAL REPORT ON 1987 MATHS INVESTIGATION

The aim of this report is

: to describe the Maths test given to the students in the Mt. Druitt Longitudinal Study in 1987,
: to describe and discuss the results,
: to analyse the errors made by the students,
: to analyse the responses to items common to the 1986 and 1987 tests
: to suggest some possible interventions to address the errors made by the students.

The Purpose of the Test

The test aimed at providing a means of comparing the achievement of the students both within and across the Year 9 sample as well as providing a basis for comparison of maths achievement over a period. In addition the test, through the analysis of errors, was to be a diagnostic tool for use by the teachers.

Method

Subjects

The subjects sitting for the test were Year 9 students who had participated in the Mt. Druitt study in previous years as well as some additional students joining the study in 1986. The students attended either of the three Mt. Druitt high schools or the two comparison high schools numbering 583 in Mt. Druitt schools and 80 in the contrast schools, giving a total of 663 students.

Design

To fulfil the purpose listed above the instrument needed:

1) to provide a means of assessing student performance and detecting student weaknesses;
2) to have the potential to distinguish between the ability of the students in the different sites; and
3) to possess a range of items of varying difficulty, a number being items repeated from the 1986 testing, others being added at later testings to give a coverage of a range of difficulties appropriate to the age of the students.

Instrument

The instrument was compiled using items from the item bank of the ACER Maths Profile Series, the same bank used for the earlier 1985 (Low, B. and Eltis, K. (in press) and 1986 (O'Brien, P. and Richardson, A., 1987) tests. The Maths Profile Series is a bank of questions which is designed to measure students' mathematical development throughout their schooling. This is achieved by way of a common scale of measurement. The items in the Series are grouped by their difficulty and the students are grouped according to their ability. The common scale is marked in equal units called "brytes" which indicate the difficulty of the item and the ability of the student for which it is a suitable item. Increasing

brytes indicate increasing item difficulty and student ability. This item bank and scaling procedure provide a means of comparing the performances of students at any one time as well as providing a means of comparison over time.

The age norms (see Appendix 5), are based on experimental data collected by Collis (1975) and Wines (1975) and from ACER trial tests in a number of Victorian, New South Wales and Queensland schools. These norms are suggested as guides only because the trial schools, whilst possessing varying backgrounds, are in no way a representative sample of schools.

The items chosen for inclusion in the 1987 test were from four major areas; Measurement, Operations, Number and Space (Table 1).

Items covered the content and process areas of the four topics along the continuum of difficulty with bryte levels ranging from 45 to 58. These are lower than those suggested in the ACER Handbook for Year 9 students as the results from the 1985 and 1986 maths testing indicated the suggested norms were too high (see Appendix 1-4 for the breakdown of the items over content area, processes and degree of difficulty and Appendix 5 for a description of the ACER Profile Series).

Table 1 **Classification of Items in ACER Maths Profile Series**

Major Area	Content Area
1. Measurement	a) Statistics, Money b) Perimeter, Area, Volume c) Measurement systems d) Time, Rates
2. Space	a) Basic concepts, Perception, Topology b) Location, Distance, Angular relations c) Shapes, Solids (properties & relations) d) Spatial transformations
3. Number	a) Natural numbers, Integers b) Indices, Logarithms c) Ratio, Percentage Rates d) Rational numbers
4. Operations	a) Small numbers b) Large numbers c) Pronumerals

The items are divided into three process areas which summarise the learning activity involved for each item. These are:

a) basic processes relating to knowledge of basic concepts, the ability to perform operations, to classify and to order objects;

b) understanding processes involving relevance, interpretation and communication; and

c) application processes in problem solving, reasoning and cognitive creativity.

Two tests, one of 30 items and one of 24 items, were trialled in two schools, one similar to the Mt. Druitt sub-sample and one to the Contrast sub-sample, on a total of 60 students and the results showed a spread consistent with the aims of the test.

As well the two tests were given to the teachers from the sample schools for examination and comment. This resulted in two items being rejected because one item was considered by the teachers to be too difficult and the other involved subject matter which had not been formally treated. These two items were replaced, and the resulting tests were of 30 and 24 items respectively with the former comprising the Measurement and Operations items (15 of each) and the latter comprising the Number and Space items (12 of each). It was expected that both tests would be of 30 minutes duration in order to fit comfortably into the normal 40 minute mathematics period. To enable a comparison between the achievement of the students in 1986 and 1987, twenty-four items, which were found difficult by the students in 1986, were included in this test (see Appendices 1-4 for these questions).

The tests (see Appendices 7 and 8 for the Test scripts) were administered on separate occasions, within one school week, by class teachers during April to all students of Year 9 who were in the study. Teachers had been instructed in the administration procedures and were given a copy of written instructions for administering the test (see Appendix 5).

The total number of students tested was 567 of whom 489 were from the Mt. Druitt schools sample and 78 from the contrast schools. The number of females was 270 and males, 259. When the tests were analysed it was found that a number of students had not completed both tests and hence their results were omitted from the summary of achievement results giving final sub-sample sizes of 403 and 75 with 239 females and 232 males.

RESULTS

Mathematics Achievement

To determine whether any differences existed within and across the schools in the study a number of analyses were performed on the total sample, the subsamples (Mt. Druitt and Contrast schools), the individual schools and classes all of which were separated by gender for both the total score and subscores. The results are summarised in Table 2.

Table 2 **Differences in Mathematics Achievement**

Table 2A

Group	Number	Achievement Scores	
		Total Score (max=54) mean	s.d.
Total Mt Druitt Subsample	403	23.50	10.24
School 1	140	23.21	9.65
School 3	126	24.40	11.23
School 5	137	22.97	9.90
Total Contrast Subsample	75	36.41	8.70
School 2	43	34.51	9.41
School 4	32	38.97	6.99
Females	239	24.75	10.48
Males	232 *	26.28	11.63
TOTAL	478	25.53	11.06

Table 2B

Group	Number	Achievement Scores			
		Measurement (max=15) mean	s.d.	Operations (max=15) mean	s.d.
Total Mt Druitt Subsample	403	6.59	3.07	6.36	3.61
School 1	140	6.54	2.91	6.19	3.46
School 3	126	6.54	3.24	6.67	3.91
School 5	137	6.70	3.08	6.26	3.47
Total Contrast Subsample	75	9.73	2.77	10.43	3.55
School 2	43	9.05	2.86	9.88	4.02
School 4	32	10.66	2.39	11.17	2.70
Females	239	6.82	2.99	6.89	3.75
Males	232 *	7.32	3.45	7.12	4.02
TOTAL	478	7.09	3.23	7.00	3.89

Table 2C

Group	Number	Achievement Scores			
		Number mean	(max=12) s.d.	Space mean	(max=12) s.d.
Total Mt Druitt Subsample	403	5.10	2.55	5.44	2.93
School 1	140	5.08	2.51	5.41	2.94
School 3	126	5.40	2.66	5.79	3.05
School 5	137	4.86	2.47	5.15	2.80
Total Contrast Subsample	75	7.56	2.09	8.69	2.42
School 2	43	7.61	2.17	7.98	2.49
School 4	32	7.50	2.02	9.66	1.96
Females	239	5.37	2.58	5.67	2.95
Males	232 *	5.61	2.71	6.23	3.22
TOTAL	478	5.49	2.64	5.95	3.09

* 7 students did not indicate gender

Results reported in Table 2 indicate that, as expected from previous testing, the Mt. Druitt subsample of students achieved a significantly lower average score, compared to the contrast subsample, on all subscores and on the total score. The difference between the Mt. Druitt and Contrast school means of the total scores of approximately 13 is educationally important as well. It is to be noted that this difference in the means has not increased when compared to the 1986 results. The greatest difference in the mean total score of individual school groups peaked at 16 (between Schools 4 and 5), again educationally very important and again no worse, in fact, better than in 1986.

A series of t-tests was performed to determine significant differences between pairs of schools with respect to total scores and subscores. Table 3 indicates the pairs of schools between which there were significant differences on mean total scores.

Table 3 **Multiple Classification Analysis - School Group Differences on Total Scores**

	School 1	School 2	School 3	School 4	School 5
School 1					
School 2	*				
School 3		*			
School 4	*		*		
School 5		*		*	

* denotes pairs significantly different at .001 level

The results of this analysis on Total Scores indicate that:

1) both contrast schools were significantly different to the Mt. Druitt schools and this difference existed in all subscores;

2) the contrast schools were significantly different to each other at $.02 < p < .05$ as for 1986;

The lack of any difference between the three Mt. Druitt schools shows a return to the findings of the earlier primary maths testing (Houghton, 1981; Houghton and Low, 1982) in which the sample presented as homogeneous. Contrary to the findings in the Year 8 testing (O'Brien and Richardson, 1987) there were no significant differences amongst the Mt. Druitt schools in any of the subscores. The reason for this may be that one of the lower stream classes at School 1 did not sit for the test in 1986, but did in 1987. Hence, the results for School 1 in 1986 were abnormally high. The drop in performance of School 1 in 1987 should not be viewed with concern.

Progress in Mathematics Knowledge - Achievement "Gaps"

A growing divergence between the performances on standardised tests of disadvantaged youngsters and those of their non-disadvantaged peers has been frequently noted. This means that the longer the disadvantaged youngsters stay at school the further, relatively, they fall behind. In Table 5 are reported standard deviation unit differences; measures of relative performance. Whilst it is clear from the Table that across the primary years the Mt. Druitt students learning rate was somewhat better than the contrast schools' students (see the final column) a significant gap opened up with transition into high school. Fortunately the gap closes slightly in Year 8, indicating that, once again, the Mt. Druitt youngsters are learning at a slightly better rate than the contrast students. The increase in the achievement gap occurred with a change in maths testing style as well as with the primary school/high school transition. It would be fair to say that the high school maths tests (Years 7, 8 and 9) are cognitively more demanding than those of earlier years. As could be suggested from the primary school performance, the Mt. Druitt students appear in high school to be somewhat slower in rate of intellectual development. Clearly though, the Year 8 results indicated that they were catching up as they did in primary school. The decrease in the standard deviation unit confirms that. From the Year 9 result it appears that, whilst no further bridging of the gap has occurred, the Mt. Druitt students have maintained the same relative performance as in Year 8.

Table 5 **ACER Mathematics Achievement Results Expressed as Standard Deviation Unit Differences for Mt. Druitt Compared to Contrast School Children**

Year	Grade	Contrast n	Contrast $\bar{X}$	Contrast s.d.	Mt. Druitt n	Mt. Druitt $\bar{X}$	Standard Deviation Unit Difference*
1981	3	307	89.96	10.71	577	73.99	-1.49
1982	4	275	59.17	12.57	483	46.37	-1.02
1983	5	201	71.47	13.95	433	59.84	-0.83
1984	6	183	70.62	11.13	379	63.12	-0.68
1985	7	54	26.96	2.09	383	22.11	-2.32
1986	8	79	38.21	9.11	487	25.14	-1.44
1987	9	75	36.41	8.70	403	23.50	-1.48

* s.d. unit difference = $\dfrac{\bar{X}\text{ (Mt Druitt)} - \bar{X}\text{ (Contrast}}{\text{sd (Contrast)}}$

Table 5 is best read in two parts, the first from 1981 through 1984 and the second from 1985 onward. This break coincides with the move to high school and to a difference in approach to testing maths achievement.

Gender Differences

Throughout the primary school years (1981-1984) of the Mt. Druitt Study the results of the mathematics testing have shown that in the Mt. Druitt schools the females have achieved superior results on maths tests.. The reverse occurred at the Contrast schools. It was appropriate to examine whether this pattern is continuing into secondary schooling. Accordingly, for the 1987 results, an examination of each school's mean score and subscore by sex was undertaken.

The results are displayed in Table 6 indicating the relative performance of males and females in the test. Whilst the results for the Contrast schools maintain the superiority of the boys, of particular interest are the results for the Mt. Druitt subsample. There were no significant differences between the performances of the males and females within Mt. Druitt schools. School 1 demonstrated a pattern of male superiority on all but one measure (the same measure as in 1986) unlike the other two schools. The pattern of superiority for School 3 was changed markedly from 1986 when the females were significantly superior in one subscore, and superior in one other as well as the total score. Whether the results are signalling a change in the overall performance of females which will ultimately lead to the pattern found in the Contrast schools remains to be seen.

Table 6 **Gender Differences in Performance in Subscores and Total Score**

	Mt. Druitt School			Contrast School	
	1	2	3	1	2
Measurement	=	√	√	√√	√
Operations	√	=	=	√	=
Space	√	=	√	=	√
Number	√	*	=	√	√
Total	√	=	=	√	√

* indicates females superior
√ indicates males superior
√√ indicates males significantly superior
= indicates no difference.

ANALYSIS OF LONGITUDINAL CHANGE IN SPECIFIC ITEMS

As stated earlier, one of the purposes of this test was to provide a comparison in achievement over a period of time. To do this, twenty-four items, from the 1986 test which had the greatest potential for yielding improved performance, were included in the 1987 test. The responses of the students who participated in both tests were analysed to ascertain the extent of the improvement. The results are displayed in Table 7 which shows that, for any one item, some students improved in performance, others declined whilst still others

remained unchanged. The major feature of the Table is that it enables the identification of questions which have become more readily solved and those with which students are still encountering difficulty.

The table is read by analysing the percentage of students who continue to make a mistake in an item (incorrect 1986/87) and those who decline in performance (correct 1986, incorrect 1987) in comparison with those who improve (incorrect 1986, correct 1987). Doing this shows that questions 21 and 35 had the greatest improvement rate which demonstrates a slight upward shift in the performance of the Sample for these two items. Questions 9 and 45 are cases in which the percentage of students who continue to make a mistake is approximately the same as those who improve. There is a majority of questions, for example questions 6, 8 and 49, in which a large number of students are continuing to err or have declined in performance, and which remain beyond the level of the Sample.

Table 7 **Change in Performance for Questions Common to 1986 and 1987 - Whole Sample**

Question/1987 Topic	Incorrect % '86-'87	Correct '86 (%) Incorrect '87	Incorrect '86 (%) Correct '87	Correct '86 (%) Correct '87
2 Measurement	35.2	20.1	18.4	26.4
3 Measurement	26.8	13.8	23.9	35.4
6 Measurement	42.5	15.1	23.8	18.6
8 Measurement	48.8	13.2	23.8	14.3
9 Measurement	23.1	16.8	24.2	35.9
18 Operations	41.5	11.2	16.4	30.9
19 Operations	41.5	9.5	19.7	29.4
20 Operations	44.3	10.6	22.0	23.1
21 Operations	25.5	13.8	27.4	33.3
22 Operations	45.1	10.1	22.5	22.2
26 Operations	27.2	18.8	20.1	33.9
27 Operations	27.2	24.4	14.9	33.5
44 Space	24.6	19.2	19.0	37.1
43 Space	26.8	14.0	17.7	41.5
45 Space	29.6	15.1	28.7	26.6
47 Space	22.5	21.4	21.2	35.0
49 Space	39.3	22.0	18.4	20.3
53 Space	33.0	25.5	14.0	27.4
31 Number	28.9	17.9	18.8	34.3
34 Number	41.7	19.0	25.7	13.6
35 Number	21.1	11.0	23.8	44.1
37 Number	38.0	13.8	22.9	25.3
38 Number	33.9	16.8	25.7	23.5
42 Number	43.0	16.6	17.9	22.5

It is important to note that Table 7 reveals a majority of question in which there has been a net increase in performance. Two items (questions 44 and 47) have shown no change whilst four items (questions 2, 27, 49 and 53) have shown an overall drop in performance. Question 34 is of particular interest because, whilst there is a high rate of improvement (25.7%), there is a statistically significant number of students who have declined in performance. Whilst an error analysis is included in this report it is appropriate to discuss here the possible reasons for this result. The correct solution for the question is C $\frac{210 \times 7}{3}$ cents.

A breakdown of the responses given over the two years (ignoring no responses) appears in Table 8.

Table 8 **1986 and 1987 Responses to Question 34 (1987) Test)**

		1987 Response (%)				
		A	B	C	D	Total
	A	5.62	3.46	7.99	3.67	20.74
1986	B	4.54	4.54	9.94	5.62	24.64
Response	C	5.18	4.54	13.61	7.56	30.89
%	D	2.16	3.24	4.75	2.16	12.31
	*	1.30	0.43	3.02	0.86	
Total		18.90	16.21	39.31	19.87	

* no response in 1986

From the Table it can be seen that of the students who declined in performance, the majority chose option D which was the second most favoured response. Response D was $\frac{210}{3x7}$

It could be argued that students in evaluating response D would obtain the same result as for response C by virtue of performing the calculation as 210 ÷ 3 x 7 as opposed to 210 ÷ (3 x 7). Without interviewing the students who chose D as their response it is not possible to determine the number of students who did this but based on the research of others (Hart, 1984; Bell et al, 1984), it is possible to suggest that whilst some students are unable to formally express the correct solution they are, nevertheless, able to calculate the correct solution. Consequently this decline in performance may not be as significant as it at first appeared and this question may have been the most successfully answered in terms of calculating the correct answer as opposed to correctly stating how the answer is obtained.

Of further interest is the extent to which students make the same error in both years. This can provide a measure of the extent to which remediation or reteaching may have been effective, or, as Collis (1975) suggests, a measure of the cognitive development of the students. The analysis of errors which have been made in both years by the same students appears in Table 9.

Table 9 **Analysis of Same Error Being Made in 1986 and 1987**

Item (1987)	Incorrect in both years as % of sample	Same Error in both years as % of sample
2	35.2	9.9
3	26.8	15.5
6	42.5	15.3
8	48.8	17.1
9	23.1	6.0
18	41.5	17.7
19	41.5	19.2
20	44.3	14.0
21	25.5	7.6
22	45.1	18.4
26	27.2	7.8
27	27.2	10.8
44	24.6	11.7
43	26.8	18.1
45	29.6	13.0
47	22.5	7.3
49	39.3	18.6
53	33.0	12.5
31	28.9	8.0
34	41.7	12.3
35	21.2	6.9
37	38.0	13.0
38	33.9	12.1
42	43.0	12.3

It is clear from the Table that the proportion of students making the same error in both years is less than half, with the exceptions of Items 3 and 43. This suggests that the students have changed, for some reason, the strategy involved in solving the particular questions. Of course the unfortunate thing is that, in changing strategies, they have made a different error. The worth of an analysis such as this particular one is dependent upon the use to which it is put by the teachers. It is possible to trace the path of errors made in consecutive years. For example, Question 18 $7 - 4 = \Delta - 7$ where the value of Δ is to be chosen from A11, B10, C4, D3. From Table 9 it is known that 41.5 percent of the sample were incorrect in both years and that 17.7 percent made the same error in both cases. Do the errors made by the remaining 24 percent follow any pattern? To determine whether this is the case the range of responses to this question are taken from the overall analysis and are shown in Table 10.

Table 10 **Analysis of Errors for Question 18**
1987 Response (%)*

1986 Response (%)	A	B**	C	D
A	0.86	3.67	1.30	0.86
B	3.24	30.89	4.54	1.94
C	2.81	8.42	14.69	7.13
D	0.86	3.89	4.10	2.16

* No response category has been omitted
** Correct answer.

From the Table it can be seen that response C was the most frequent error in both years. The important feature of this table is that in relation to the students who chose C in 1986, as well as a large proportion choosing C, there was a shift to response D in 1987. Thus the student has moved from matching the numbers on both sides of the equation to calculating the result of the operation on the left hand side. With this information the teacher could address the two matters raised here, that of over-generalising the commutative law in which case C has been chosen, and that of the student regarding the equality sign as implying an operation rather than being regarded as an equivalence.* It is beyond the scope of this report to perform detailed analysis of this nature, however the facility is available for teachers who seek this information.

ERROR ANALYSIS

An analysis of the students' errors was undertaken to provide teachers with a means of identifying conceptual problems, computational problems and other error types with a view to their providing remediation. For each student a total score and four subscores were furnished to their mathematics class teachers as well as a breakdown of each student's response, thereby providing a diagnostic tool for the teacher. These individual results are not included in this report for reasons of confidentiality. As well a breakdown by school, class and subscores was given to each teacher. These measures were designed to provide the teachers with as useful a diagnostic tool as possible.

The second aspect of the analysis of errors was the categorising of types of errors made by the students. In this analysis only those errors which represented a large proportion of the total error for a particular question were analysed. In some cases it was not possible to determine the strategy used by the students in making particular errors.

Error Strategies for Measurement Items (Questions 1-15)

There appear to be six types of errors which occurred in this subdivision of the Maths test:

1) Incorrect calculation: in this case the students applied the correct operation but made an error on computing, e.g Question 6 where the problem reduced to calculating 50-17 and students answered 23.

2) Interpretation of the given diagram: in this case the orientation of the student led to an incorrect response
   e.g. Q5 where the students who chose response B did not realise that by passing "O", 10 kilograms had been weighed.

3) Interpretation of the question: in this case the student is unable to isolate the problem
   e.g. Q12.

4) Knowledge of units of measurement: in this case the inability to place items in relative positions has led to error
   e.g. Q15.

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* For a detailed treatment of the various interpretations see Keiran, 1980.

5) Conceptual: in this case the error was through lack of knowledge of the particular idea being tested
e.g. Q13 where the "acute" angle is asked for and a student responds by giving the "obtuse" angle.

Table 11 summarises these errors.

Table 11 **Error Analysis of Measurement Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of * Total Errors 1	2	3	4	5	No. Response %
Units of measurement	1	80	19	47	53				1
	5	46	53	60			23	17	1
	10	56	42				34	65	2
	15	14	82				100		4
Rates, time	6	42	57	14		68		16	1
	7	57	42	56	43				1
	8	39	60			74	26		1
	12	40	59		100				1
	13	26	72	22				78	2
Perimeter, Area, Volume	3	60	38	30	70				2
	4	61	37	25	24	50			2
	11	28	69		100				3
	14	42	54	100					4
Statistics, Money	2	45	54	100					1
	9	61	38	30		69			1

** Does not include "No Response"
* Code for error strategy
1 : incorrect calculation
2 : interpretation of diagram
3 : interpretation of question
4 : knowledge of units
5 : conceptual

Error Strategies for Operation Items (Questions 16-30)

In this area seven error strategies were identified:

1. The student ignores extra terms and/or performs one calculation on one side ignoring the other side.
e.g. Q16 $(12 \div 2) \times 2 = 6$ (ignores x 2)
Q21 $(240 \div 15) = 16$ (ignores 480)

2. The student matches numbers or pronumerals in both expressions or ignores differences or overgeneralises the commutative law
e.g. Q18 $7 - 4 = 4 - 7$
Q20 $(24 \div 6) \div 2 = 24 \quad (6 \div 2)$

3. The student performs only part of the operations required and obtains a correct answer for this part.
e.g. Q17 $4 + 5 = (4 + 6) + (5 + \Delta)$ - here the students who gave -1 as their response have calculated $(4 + 6)$ and then subtracted 1 to obtain the left hand side

4. Zero misunderstanding : the student has thought that adding zero changes a number or multiplying by zero gives something
e.g. Q27 w x o = w.

5. Unit misunderstanding where multiplying by 1 gave something else.
e.g. Q26 y x 1 = 1.

6. The student performs an incorrect operation
e.g. Q19 8 : 4 = 32.

7. Conceptual error.

Table 12 summarizes these operations errors.

Table 12 **Error Analysis of Operations Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of * Total Errors						No. Response %
				1	2	3	4	5	6	
Small numbers	16	81	16	48					33	3
	17	46	51		72	27				3
	18	47	50	27	54					3
	19	49	47	32	50				19	4
	20	47	48	23	50	27				5
Large numbers	21	61	34	54					31	5
	22	46	49	29	49				22	5
	23	45	50		65					5
	24	30	64						76	6
	25	31	61		76	24				8
Pronumerals	26	53	39					42	57	8
	27	50	42				81		19	8
	28	25	67	26					73	8
	29	21	71						100	8
	30	33	59		85					8

** does not include "No Response"

* Code for error strategy

1. Inaccurate calculation
2. Wrong operation
3. Order of operation
4. Zero misunderstanding
5. Number matching
6. Ignores extra terms
7. Unity misunderstanding

It is worthwhile to examine the errors made in questions which have the same structure but which have factors which could affect the success of the students. Collis (1975) found that the incidence of errors increased as the components of the questions went from small numbers to large numbers to pronumerals. In the Operations test the following pairs of questions had the same structure but differing components, adjacent to them are the success rate for each question.

Question 18,	47% and	Question 22,	46%
Question 19,	49% and	Question 23,	45%
Question 20,	47% and	Question 25,	31%

It would appear then that the findings of Collis have been supported only marginally for the first two cases but for the third case the difference is marked.

Error Strategies for Number Items (Questions 31–42)

Four general error strategies were defined for these items:

1. Incorrect operation: in which case the correct data was used with an incorrect operation to obtain an incorrect response
   e.g. Q34 $\frac{210 \times 3}{7}$ instead of $\frac{210 \times 7}{3}$

2. Incorrect calculation: in which the data was used correctly but the answer was incorrect by a certain factor or by a unit.
   e.g. Q37 16.6 + 7.8 = 23.14

3. Conceptual: in which the student could not obtain the correct solution because of lack of knowledge of terms, processes or measurement units.
   e.g. Q12 where students were unaware of relative position of fractions.

4. Interpretation of question: in which case the incorrect answer was obtained by the wrong reading of the question
   e.g. Q7 where, in obtaining response B the tudent has interpreted the question to read "... risen to 16.6C" as opposed to "risen by 16.6C".

Table 13 summarizes these errors.

Table 13 **Error Analysis of Number Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of* Total Errors				No. Response %
				1	2	3	4	
Natural nos, integers	33	67	32	100				1
	36	32	65			100		3
	38	51	48	25		75		1
Ratio, rates, percentage	32	65	34				50	1
	34	39	59	72		28		2
	35	69	30		100			1
	40	35	63	17		83		2
	41	35	63	31	21		36	2
Rational numbers	31	54	45		67	33		1
	37	46	53	19	32		46	1
	39	32	67			100		1
	42	40	58			100		2

* Code for Error Strategy
1. Incorrect operation
2. Incorrect calculation
3. Conceptual
4. Interpretation

** Does not include "No Response"

Error Strategies for Space Items (Questions 43-54)

In this area four error types were identified:

1. Visualisation: in which case students did not see the "whole" of the problem or could not match the parts to the whole
   e.g.Q49 - not seeing the "whole" of the problem
   Q46 - not matching the parts to the whole.

2. Orientation: in which case students could not place themselves in the correct position to determine the correct response
   e.g.Q45.

3. Topological: in which case students could not complete paths or patterns to obtain the correct response
   e.g.Q47.

4. Conceptual: in which case students were unable to obtain the correct response because of incorrect matching of visual stimulus with concept
   e.g.Q53 where response A and D were given because the lines are of different lengths.

Table 14 summarizes these errors.

Table 14 **Error Analysis of Space Items**

Objective	Item No.	Success Rate	Error** Rate %	Error strategy as % of * Total Errors				No. Response %
				1	2	3	4	
Perception,	47	57	40			100		3
topology	48	36	61		100			3
	54	33	62	100				5
Location,	51	30	66		31		69	4
distance	44	59	39		100			2
shapes, solids	43	57	41	74				2
	46	26	71	100				3
	52	37	59				100	4
	49	39	58	100				3
	50	53	43				100	4
	53	42	54		25		75	4
Spatial transfer	45	55	43		100			2

* Code for Error Strategy
1. Visualisation
2. Orientation
3. Topological
4. Conceptual

** Does not include "No Response"

Discussion of Error Analysis

Before discussing these analyses generally it must be pointed out that certain assumptions are being made in obtaining these figures for the error analysis, not the least of which is that the students, in responding, were selecting the responses they felt most appropriate. It is impossible to isolate those cases where students have answered randomly. The other major assumption is that the tests were administered in such a way that no collusion was possible between students in making repsonses. (In fact, it is probable that either one or both of the above events did happen, however, it cannot be controlled.) The large number of poorly answered questions (see Table 15) may be due to the norms, as suggested by ACER, being unsuitable for students in Sydney. This statement refers, not only to the Mt. Druitt schools where the expectation is for poorer results, but to the Contrast schools where the results do not match those that would be expected from this relatively able group. The ACER Handbook states that the age norms given are only to be used as guides and that it may be necessary to develop "local" norms. This warning is reinforced when it is noticed that of the thirty-three schools trialled by ACER in developing the test, only four were New South Wales schools. For the Number test, Ibrox Park Boys High School and North Sydney Girls High School (a selective school) were used while for the Measurement Test Maroubra Bay High School and Pittwater High School students' responses were included. There are more than three hundred high schools in New South Wales. A sample of four probably does not represent a reliable sample.

The items in the Mt. Druitt Longitudinal Study test were selected using these "high" norms and the concomitant level of difficulty of items may have affected the attitude of the students towards the test.

Whilst there was variation between school groups with respect to answers to particular questions (see Table 15 for a summary of the most poorly answered questions) there are some generalisations that may be made. Firstly, there was less success for questions in the Operation area which involved large numbers. Secondly, there was a propensity for students to respond to questions such as Question 19: $8 \div 4 = \Delta \div 8$ by replacing Δ by the missing number from the left hand side. In this particular case, the response $\Delta = 4$ was given by 22 percent of students whilst the correct response was given by 46 percent of the students.

A further major cause of errors may be associated with the student's inability to analyse the question before adopting problem solving techniques, if they possess them. This occurred largely with items in the Number and Measurement categories where questions were not read correctly. It must be added that the researchers are cognisant of the lower reading levels at the Mt. Druitt high schools and efforts were made to include questions which were not too difficult to read. At the same time, however, it must be pointed out that mathematics involves more than performing operations on numbers and that reading is an integral part of problem solving in mathematics.

Table 15 suggests that particular school groups are weaker in particular topic areas. For example School 5 had the lowest percentage of correct responses on the badly answered Number questions. These results may be due to curricula emphases or programming of the subject. The latter reason should be able to be discounted as the class teachers examined the test and items were withdrawn if subject matter had not been dealt with. Further analysis of this matter would be necessary before any firm conclusions may be drawn. However, the information should provide class teachers with the stimulus for critical appraisal of teaching methods.

Table 15 **Summary of School Group Errors**

Topic/Question		Percentage of Students Correct*					
		Total	School 1	School 2	School 3	School 4	School 5
Measurement	2	45	41	61	39	84	41
	5	46	40	67	41	75	43
	6	42	35	63	41	84	36
	8	39	37	50	37	56	36
	11	28	25	39	27	44	27
	12	40	33	52	38	69	40
	13	26	22	30	23	69	22
	14	42	42	52	38	69	39
	15	14	11	13	7	13	23
Operations	17	46	41	72	41	94	38
	18	47	39	72	45	84	42
	19	49	41	67	48	88	44
	20	47	40	74	48	91	34
	22	46	38	72	45	81	40
	23	45	34	65	39	91	47
	24	30	23	57	35	59	18
	25	31	22	63	31	63	23
	28	25	21	33	22	31	29
	29	21	18	33	13	34	25
	30	33	24	57	32	63	28
Numbers	34	39	39	41	42	72	31
	36	32	30	61	26	56	25
	37	46	44	68	48	94	35
	39	32	25	57	30	53	29
	40	35	28	61	30	84	28
	41	35	25	39	40	78	29
	42	40	40	55	34	84	34
Space	46	26	28	30	24	34	22
	48	36	43	41	36	44	26
	49	39	36	70	38	59	31
	51	30	24	41	34	50	27
	52	39	34	66	36	56	37
	53	42	38	66	36	81	36
	54	33	30	59	33	38	26

* applies to most poorly answered questions (less than 50% correct on total)

Interpretation of the "No Response" Category

In attempting to explain the total lack of a response to a specific item, which ranged from less than one percent to eight percent of the total responses for a particular item, it is not possible to determine whether the students could not or would not answer the question. The fact that the number of no responses increases towards the end of both tests may suggest that test fatigue could be in evidence. Observation of students during the tests and/or follow up interviews would enable more conclusive statements.

Summary of Findings

1. There are significant differences in achievement between the two subsamples and between schools within the subsamples on both total score and subscores. The Mt. Druitt and Contrast subsamples were significantly different on all subscores and total scores.

2. The analysis of errors revealed common error strategies within and across the subsamples. The major factors affecting errors appear to be: the interpretation of the question, the relative size of numbers and the position of the symbol whose value was to be determined.

ERROR ANALYSIS AND IMPLICATIONS FOR TEACHING : SOME POSSIBLE INTERVENTIONS

Following the responses made by students to the questions in the 1987 Mathematics test, what follows are some suggestions for general and specific strategies which may be of assistance in dealing with the errors. It is hoped that these suggestions might be useful and feedback to the research team from teachers will be sought. Several references which deal with error strategies and which may provide more help are identified.

Language and Mathematics

A concern of many of the teachers in the study was that some of the test items were too "wordy" and were not testing Mathematics and hence should be omitted. However, rather than avoiding these types of questions, it is important that students should come to grips with them. It is essential that language skills necessary to mathematics be developed. The Cockcroft Report (1982:91) suggests this would be best achieved through clear, unambiguous explanation, joint discussion and by encouraging students to talk and write about their investigations.

There are two major aspects of language relevant to this test. Firstly, there is the search for verbal cues made by students in an attempt to link an operation with the written word. Unfortunately, this search often leads to errors because of different operations being possible for the same word or because of the student wrongly interpreting the cue word.

An example of the first case would be the following situation:

1. Peter has \$5 and Jenny has \$3 more than Peter. How much money does Jenny have?

2. Peter has \$5 and Jenny has \$8. How much more does Jenny have?

Here the word "more" has associated with it two different operations. It is essential that students learn to determine the context of the particular cue word.

An example of the second case occurs in Question 37 where many students interpreted the word "by" as "to" and subtracted the two temperatures instead of adding. Similar problems occur in cases such as "a fall of" compared with "a fall to", "to 12th place" compared with "from 12th place", "a bearing of A from B" compared with "a bearing from A to B".

The second aspect of language is that, upon finding the cue words, it is important that students refer back to the question as a whole.

e.g.1. In Question 8 the majority of students having found the cue words "name of the month" and "starts with A" did not relate this information to the word "before". Hence 23 percent of students obtained "August" as the answer whilst 37 percent had the correct response.

e.g.2. In Question 6, the word "remaining" was not correctly identified and 37 percent of students gave as their answer the amount of fuel used (40% were correct).

Thus greater exposure to, and discussion of, questions involving the use of language is recommended whilst the minimisation of language usage in questions is discouraged.

Rational Numbers

Whilst students should have experienced the use of concrete materials in learning about fractions prior to Year 9 the learning appears not to have been effective. It is recommended that students actually see, for example, (so that they can later visualise and generalise) a half of the apple being divided into three parts and these pieces being related to the whole apple. This approach also reinforces the relative size of fractions and their relative position on the number line as was tested in Question 42 where from the number line given

students were asked which of 1/2, 3/4, 4/5, 1/8 is a point in the shaded section (38% were correct whilst 25% chose 3/4).

An approach to teaching the relationship is to actually generate the misconception, and then discuss the implications of this using whatever aid is appropriate and to then resolve the problem, relying heavily on student participation. (This requires considerable self-control from teachers who have an almost subconscious desire to put words into the mouths of students.) This so-called Conflict Approach (Bell, 1984) can be very effective in resolving misconceptions.

A further example of using this approach, cited in Bell (1984) can be useful with students who consider decimals to be two separate integers separated by a ".". A "conflict" approach would be to have students count orally 0.3, 0.6, 0.9 ... etc. Those who have the mentioned misconception would continue 0.12, 0.15 and so on. Having students do the "counting" on a number line or calculator leads to a confrontation with the error and, hopefully, to a resolution. This could prove effective in relation to Question 39 where students were asked to choose the point representing Y from 1.2, 0.4, 0.2 and 0.04.

The majority of students chose 0.2

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Operations

Many of the errors which occurred in this section stemmed from the over-generalisation of a previously learned rule; e.g. in Questions 18, 19 and 20 many students (54%, 50% and 54% respectively) over-generalised the commutative law of addition and multiplication to subtraction and division. To overcome this, the giving of varied examples and non-examples of the rule should aid in countering this misconception. In other words, rather than have students witness that $7 + 5 = 5 + 7$ and $7 - 5 \neq 5 - 7$, students should also witness that, for example, $615.95 + 37.6 = 37.6 + 615.95$ and $615.95 - 37.6 = 37.6 - 615.95$ and that $11.1 - 8.74 \neq 8.74 - 11.1$. Too often these types of rules are introduced using "easy" or "simple" examples whereas varying the numbers could lead to correct generalisations of the rules. This would then prevent the increase in errors when one considers Questions 26 and 30 where exactly the same rule is applied yet an extra four percent of students were incorrect in the question involving large numbers.

The over-generalisation problem appears to have occurred with respect to addition and multiplication by 0 and 1, e.g.

Question 27 — 22 percent of students responded that $w \times 0 = w$ compared with 29 percent in 1986.

Question 26 — Only 53 percent of students responded that $y \times 1 = y$ (compared to 50% in 1986).

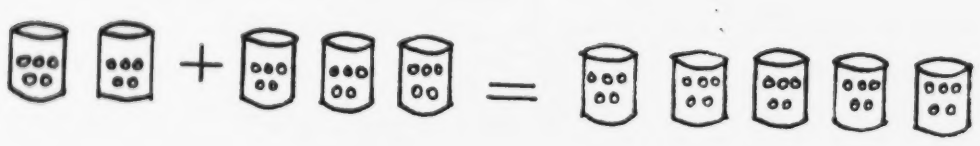
These errors, which occurred more frequently but not exclusively in the lower ability classes, could also be attacked using the conflict approach, illustrating the incorrectness of the student's answers and then the correct response being reinforced using examples and non-examples.

That more students were incorrect when a pronumeral was introduced as is the case of Questions 17 and 30 suggests that the concept of pronumerals is not clearly understood. This misunderstanding is further emphasized in Question 30 in which nine percent of students in answering $p + q = (p + r) + (q + \Delta)$ gave $\Delta = s$ as their response. It is important that students do not interpret a pronumeral as an object, an occurrence which, unfortunately, tends to be reinforced by operations such as addition of monomials; e.g. $2x + 3x$ and $2a + 3b$. Students should never be told, as they too often are, for the latter example: "since you cannot add tables and chairs, you cannot add a's and b's". This type of justification, as well as being mathematically unsound, has inherent problems. It does not address the concept of a pronumeral.

A pronumeral does not represent a "table" or a "chair", it represents a number. A more practical way of illustrating this would be to use counters and cups along the following lines, noting that the teacher has emphasised that "x" represents the same number in each separate situation.

For $2x + 3x$, the teacher arbitrarily chooses a number to represent x, say 5, without telling the students, and places this number of counters into each of five identical cups.

Hence $2x + 3x$ becomes:



The students then count the number of counters in the total and see that the answer is not five cups but five lots of the number inside each cup. It is important that the students see the counters at the end to reinforce the idea that x represents a number. (This method can be extended to make the teaching of solutions of equations of the type $5n - 6 = 3n + 12$ more meaningful and comprehensible.)

It is then a relatively simple matter to illustrate why 2a and 3b cannot be added, assuming a and b are different numbers.

The second problem with treating pronumerals as objects is that it leads to confusion in the minds of the students when multiplication of monomials is introduced. Having been told that 3a and 2b cannot be added because "you cannot add tables and chairs" the student now finds that he can multiply tables and chairs! This inconsistency and confusion is eliminated quite simply by always referring to pronumerals as numbers.

Spatial Questions

The major reason for errors in this aspect of the test probably stems from a lack of experience with manipulation of shapes and objects. However, the new Year 7 and Year 8 syllabus is addressing this matter. The DIME range of materials in both 3-D solids and Algebra are recommended for facilitating spatial perception and developing algebraic concepts in a readily understandable and enjoyable way.

Some Concluding Remarks

Some general observations may be made concerning the analysis of errors and learning interventions.

Firstly, as the Cockcroft Report (1982) and Hart, K.M. (1984) suggest, teachers of mathematics need to analyse the processes that students use in solving problems so that misconceptions can be identified and corrected. This can be done by examining scripts and/or by small or large group discussion of the questions and students' suggestions for their solutions. This process, whilst far more demanding and time consuming, is of much greater importance than that of determining whether a response is correct or incorrect.

Secondly, for developing understanding of mathematical concepts there is little to be gained from giving students a set of problems involving the same concept with different numbers in different contexts (as is the case in most mathematics text books). It is preferable to concentrate on changes of the structure in a fixed context or on the transfer of structure to different contexts (Bell, 1984). This can be done by asking pupils to match questions of the same type to write questions of a similar type but with a variation in context, or to provide a stimulus and have students compose questions),

e.g. the calculation of "speed" using $S = \frac{D}{T}$ is the same type of question as calculating the "price" using $P = \frac{\text{Total Cost}}{\text{Number}}$

Most textbooks treat these two processes under different headings in different parts of the book when they are clearly related. Teaching generic principles rather than merely aiding fluency is to be preferred as an instructional strategy.

Finally, it is necessary not to underestimate or underutilise the presage skills that students possess, nor to ignore what might appear as "ad hoc" methods in solving problems. Research indicates that in solving problems, primary and junior secondary students rarely use formal methods of solution. They do not identify the type of problem and apply **the** method of solution (Cockcroft, 1982; Hart, 1984; Bell, 1984). The implication is that problems should be developed from ones with which students can cope to more complex ones which do not involve steps which require memorisation or which lead to complicated equations. There is perhaps much truth in what Hart (1984:83) states when she says:

> that the best time to teach a child a new skill is when it is needed because the child is faced with a situation that cannot be dealt with using previously employed methods ... the (children) would retain and use "child methods" as long as they were adequate and more formalised procedures ... (are not used) until seen as essential.

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These references, including the Cockcroft Report, provide many illustrations of [illegible] and suggest ways of addressing the particular [illegible]. Ideas of investigation are proposed which could provide [illegible] strategies in attacking students' misconceptions.

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Test Question	Process Area
1	Understanding
2(8)*	Application
3(11)	Basic
4	Understanding
5	Understanding
6(15)	Application
7	Application
8(9)	Understanding
9(10)	Application
10	Understanding
11	Application
12	Understanding
13	Basic
14	Basic
15	Basic

** A "Psych" [illegible] Brown and Douglass (1975)

* [illegible] the 1986 test.

Appendix 2

Breakdown of Operations Items

Test Question	Test bank item/test	Brytes	Content
16	12	48	Small nos.
17	14	52	Small nos.
18 (18)*	15	51	Small nos.
19 (17)	16	50	Small nos.
20 (28)	18	53	Small nos.
21 (29)	33	53	Large nos.
22 (27)	35	52	Large nos.
23	36	54	Large nos.
24	37	55	Large nos.
25	38	58	Large nos.
26 (25)	44	47	Pronumerals
27 (23)	50	48	Pronumerals
28	51	53	Pronumerals
29	52	57	Pronumerals
30	54	56	Pronumerals

Test Question	Process Area
31 (46)*	Understanding
32	Application
33	Understanding
34 (53)	Understanding
35 (44)	Application
36	Understanding
37 (54)	Application
38 (52)	Basic
39	Basic
40	Basic
41	Application
42 (50)	Basic

Appendix 4

Breakdown of Space Items

Test Question	Test bank item/test	Brytes	Content Area	Process Area
43 (37)*	19/1	57	Shapes	Understanding
44 (33)	14/1	47	Shapes	Basic
45 (39)	23/1	50	Spatial	Basic
46	27/2	53	Shapes	Application
47 (40)	17/2	52	Perception	Basic
48	18/2	52	Perception	Understanding
49 (35)	3/3	48	Shapes	Basic
50	6/3	50	Shapes	Understanding
51	25/3	52	Location	Application
52	23/2	53	Shapes	Application
53 (42)	2/4	52	Shapes	Application
54	9/4	56	Perception	Application

The A.C.E.R. Maths Profile Series...

This series has been designed to provide a flexible system for monitoring students' mathematical development throughout their schooling.

Tests provided in the series use a measurement scale based on the Rasch approach. The Rasch scale provides a common measurement scale on which student ability and item difficulty are simultaneously calibrated, independently of any external characteristics of the students tested or the items answered, except those relevant to their interaction (ie. to the latent trait being analysed).The Rasch model of measurement of any trait is based on the idea that the odds of success by a person ona particular task depend on the interaction between the ability of the person and the difficulty of the task.eg. when a person's ability is greater than the task difficulty, then the odds of success are high. For Rasch scales the concepts of ability and difficulty are defined in terms of this concept of the relative odds of success which must be consistent across a variety of interactions between people and tasks.

The common scale used in the ACER Maths Profile Series is referred to as the MAPS Scale.This scale provides a measure of the latent variable which all the tests in the series have in common...ie. mathematical development. Items used in the series are located by their MAPS difficulty and students by their MAPS ability.The items have been located in their relative positions on the MAPS scale according to the degree of mathematical challenge that they offer students. These item measures have been called MAPS difficulties. For each set of items combined into a test, the marks obtained by students reflect the different levels of mathematical development elicited by the items offered. These total test scores can easily be converted into student measures on the common scale, thus giving the students' MAPS abilities.

The MAPS scale is marked in equal units called "brytes". Increasing brytes indicate increasing MAPS ability of students and increasing MAPS ability of items.

How can you use test results???

1) Achievement rankings and diagnostic profiles...The common scale for all tests allows one to compare student MAPS abilities. This enables achievement ranking or grading across a group of students. Another purpose is to draw up profiles for each student in the class. This could be done for different content areas and assist in determining the relative strengths and weaknesses of a student. Profiles across time may also be useful.

2) Criterion-referenced evaluation and relative mastery...The Rasch measurement approach allows one to assess the likelihood of a student with a known MAPS ability

of being able to correctly answer any item of known MAPS difficulty. Following from this, mastery levels have been devised quoted in terms of percentage success over a group of items of representing a desired objective. Criterion levels of 100%, 90% and sometimes 75% are usually specified. Where student MAPS ability equals item MAPS difficulty the even chance of success can be interpreted as 50% mastery. As MAPS ability increases relative to MAPS difficulty so does the corresponding mastery level; and conversely as MAPS ability declines relative to MAPS difficulty so do expected mastery levels.The potential for interpretation at a variety of levels makes it possible to assess a student's status in relation to a sequence of learning objectives and to identify those objectives for which continued learning activities are most likey to be of most benefit to a student. Theory suggests that most suitable learning activities are likely to be reflected by items having MAPS difficulties within the range of -5 brytes and to +5 brytes of each student's MAPS ability.

3) Cognitive levels and their implication for student learning...the MAPS scale has been approximated to the Piagetian stages. 25% to 90% mastery ranges at each cognitive stage follow the following pattern:

Cognitive stage:	25% to 90% mastery
Early concrete operational	30 to 45 brytes
Late concrete operational	40 to 55 brytes
Concrete generalization (early formal)	50 to 65 brytes
Late formal operational	6 to 75 brytes

Approximate age/school 'norms' are as follows (but these may vary according to localty and mathematical background of the student):

Mean MAPS ability of students for ages 7 to 17 years

Age	Related years of schooling	Mean MAPS ability(nearest bryte)
7 years	2	29
8 years	3	37
9 years	4	44
10 years	5	47
11 years	6	49
12years	7	5
13 years	8	51
14 years	9	53
15 years	10	55
16 years	11	58
17 years	12	65

The ACER Maths Profile Test for Year 9: Mount Druitt Longitudinal Study.

This test consists of 60 items designed to monitor students' development in four separate aspects of mathematics.The items in each section have been chosen from a longer test from the ACER Mathematics Profile Sreies. As the four samples of items have been selected to represent the original test, the results from administering each sample can be used to give a profile of attainment in the four areas.

The four areas of mathematics covered in the test are:
operations;
measurement;
numbers;
space.

The first section of the test has 15 items from the ACER Maths Profile Series-Operations Test.The Operations test was designed to assess students' ability to handle familiar operations in the real number field. Five items involve operations upon small numbers, five involve operations upon large numbers and five have operations involoving pronumerals.

The second section has 15 items from the Measurement Test. This test was designed to monitor students' development in those aspects of maths pertaining to measurement and mensuration. Four items involve statistics or money, three involve area, volume or perimeter, four involve measurement systems and four involve time or rates.

The third section has 15 items from the Space Test. This test was designed to assess students' spatial development, considering both traditional topics and some of the perceptual aspects which underlie these topics. Three items involve basic concepts, perception or topology, four involve location, distance or angular relations, six involve shapes or solids including properties and relations and four involve spatial transformations.

The fourth section has 15 items from the Number Test. This test was designed to monitor students' development with regard to number concepts, properties and relationships. Four items involve natural numbers or integers, two involve indices or logarithms, three involve ratio, percentages or rates and six involve rational numbers.

INSTRUCTIONS FOR TEST ADMINISTRATION.

Appendix 6

General:

1) The ACER Mathematics Profile Series consists of two parts which are to be administered at different times. The first part consists of 30 items and the second, of 24 items. Both tests require about 30 minutes actual testing time. An additional 10 minutes should be allowed for introductory procedures.

2) There are practice questions on the second page of both test booklets. The prime function of these is to introduce students to the method of answering. The name and significance of Δ may vary according to year level. It is suggested that for up to Year 8, Δ could be discussed as the triangle or the frame in which the numeral can be written. If students are familiar with the concept of pronumeral, Δ could be discussed as delta and the mathematical sentence as an equation.

3) During administration the sections enclosed in boxes are to be read aloud. However the text can be adapted to better suit the needs of their particular students.

4) Students should have working paper available so that test booklets are not spoiled.

Testing:

1) Direct students to fill in the section of the answer sheet with school, name, class age and birth date.

2) Direct students to the instructions on the front page of the test booklet and then to the continuing instructions on the second page. Depending on which is more appropriate for your particular class, either: (a) ask students to silently read the directions up until the practice questions, or (b) ask students to follow silently as you read the directions aloud.

3) The purpose of the practice questions is to ensure that students know how to recordanswers to multiple choice questions. Please do not show students how to perform the mathematical operations necessary for answering the questions.

Look at practice question P1. It says (12÷2)×2 = Δ . Put your hand up if you know the answer.

Obtain the correct answer from a student.

Yes, to make the sentence true, Δ stands for 12. So the letter A has been marked on your answer sheet beside P1.

Students can be referred to their answer sheets to see how the answer to P1 is recorded.

4 Now try practice question P2 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, B is the correct answer, so you should have marked the letter B on your answer sheet beside P2.

Teachers may wish to demonstrate this on the chalkboard and check that students have placed the correct letter in the correct place.

5 Now try practice question P3 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, A is the correct answer, so you should have marked the letter A on your answer sheet beside P3.

6 Now try practice question P4 yourselves, and mark your answer sheet.

After about 30 seconds obtain the correct answer from a student.

Yes, A is the correct answer so you should have marked the letter A on your answer sheet beside P4.

7 Are there any questions about this method of answering?

Answer any questions concerned with the method of answering.

8 You will have 30 minutes (or whatever time is appropriate) for this part. Work as carefully as possible but do not spend too much time on any one question. If you cannot answer a question after thinking about it, leave it and go on to the next one. You may have time to come back when you have finished the rest of the part. If you finish before time, check your answers, then close your booklet and wait quietly. Are there any questions?

If no further questions, say:

9 Turn over to the next page and start work now.

Record the time and check that all students have turned to the correct page.

10 After 30 minutes (or whatever time is appropriate), say:

Finish the question you are working on, then close your booklets.

11 As answer sheets are collected, check that the information is correct and entered clearly.

12 Check that all test booklets and answer sheets have been handed in before concluding the testing activity.

13. For the second test, please use this same model for administration.

The correct answers for the four practice questions are

P1	C
P2	D
P3	C
P4	D

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MATHEMATICS TEST*

THERE ARE 30 QUESTIONS IN THIS MATHEMATICS TEST. ALTHOUGH SOME QUESTIONS MAY NOT BE EXACTLY THE SAME AS THE WORK YOU DO IN CLASS, YOU SHOULD TRY TO WORK OUT THE CORRECT ANSWER.

PLEASE ANSWER EACH QUESTION BY CIRCLING ONE OF THE LETTERS ON THE ANSWER SHEET. PLEASE DO NOT MAKE ANY MARKS IN THIS BOOKLET.

THANK YOU FOR YOUR HELP.

* Items selected from ACER Mathematics Profile Series.

MATHEMATICS TEST

Practice Sheet

This test contains 30 questions.

In each question alternative answers are shown by the letters A, B, C, D or E. Mark your answer with a circle on the answer sheet.

The following practice questions will show you how to answer the questions on the text. When you see △ in a question you should choose the alternative to replace △ and make the sentence true.

Practice Questions

P_1

$5 \times 17 = 5 \times \triangle$

A $\frac{17}{5}$ C 17

B 5 D 85

P_2

$p + q = q + \triangle$

A r C $-p$

B p D $p - 2q$

P_3

The measure of the shaded region in this diagram is called the

A area. C circumference.
B perimeter. D volume.

Since area is the correct answer, the corresponding letter A has been marked on the answer sheet.

P_4

Which one of the following measures the longest distance?

A 1 kilometre C 100 millimetres
B 10 metres D 1000 centimetres

Please wait until you are told to begin the test.

MATHEMATICS TEST

Practice Sheet

This test contains 30 questions.

In each question alternative answers are shown by the letters A, B, C, D or E. Mark your answer with a circle on the answer sheet.

The following practice questions will show you how to answer the questions on the text. When you see Δ in a question you should choose the alternative to replace Δ and make the sentence true.

Practice Questions

P_1

$(12 \div 2) \times 2 = \Delta$

A 12 **C** 3

B 6 **D** $\frac{1}{12}$

P_2

$p + q = q + \Delta$

A r **C** $-p$

B p **D** $p - 2q$

P_3

The measure of the shaded region in this diagram is called the

A area. **C** circumference.
B perimeter. **D** volume.

Since area is the correct answer, the corresponding letter **A** has been marked on the answer sheet.

P_4

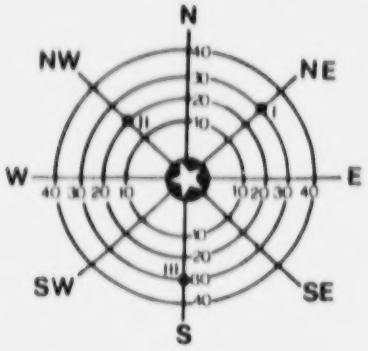
Which one of the following measures the longest distance?

A 1 kilometre **C** 100 millimetres
B 10 metres **D** 1000 centimetres

Please wait until you are told to begin the test.

1

Imagine you are standing at ✪ in the diagram.



A tree, marked by I, is 30 metres away in a NE direction and this can be written (30, NE). In the same way, II represents a tree (20, NW).
Which one of the following gives the position of tree III?

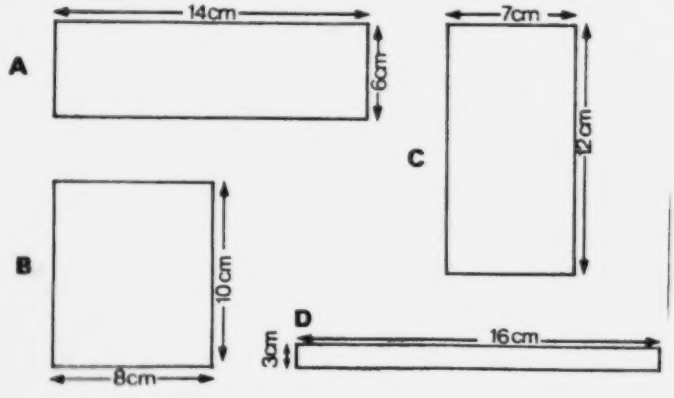
A (20, S)
B (30, S)
C (30, N)
D (30, SE)

2

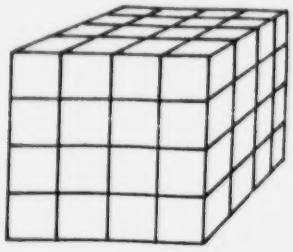
Jane asked Maria to buy her some jewellery, costing 10 000 lire, in Florence. If one Australian dollar was equivalent to 850 lire, how many Australian dollars does Jane need to give Maria to buy the jewellery?

A $12
B $85
C $90
D $117

3 Which one of the following rectangles has the longest perimeter?



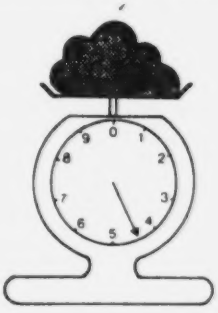
4 This figure represents a block built from 64 small cubes.



If all the small cubes from this block are used to build a solid rectangular box with a base this size which one of the following is the height of this rectangular block, in number of cubes?

A 5
B 16
C 17
D 64
E 68

5 The diagram represents a spring balance marked in kilograms, with the needle and spring set up so that at least ten complete turns are possible.



When an object is placed on the pan the needle moves twice past the zero and settles as shown in the diagram.
Which one of the following is nearest to the weight of the object?

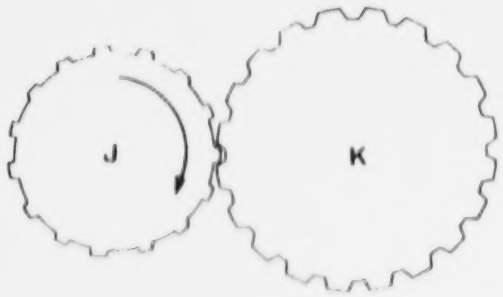
A 4.5 kilograms
B 22.5 kilograms
C 24.5 kilograms
D There is insufficient information to tell.

6

A car's fuel tank contains 50 litres of petrol, and the car uses one litre of petrol every 10 kilometres. If the car travelled 170 kilometres, the quantity of fuel remaining would be

A 10 litres.
B 17 litres.
C 23 litres.
D 33 litres.

7 J and K represent two cogwheels, with 30 and 40 teeth respectively.



If J is turning clockwise at 24 revolutions per minute, what is the direction and turning rate of K?

A clockwise at 18 revolutions per minute
B clockwise at 32 revolutions per minute
C anticlockwise at 18 revolutions per minute
D anticlockwise at 32 revolutions per minute

8 Sam's birthday occurs in a month which has 31 days. The name of the month before Sam's birthday starts with an 'A'. Sam has his birthday during

A March
B May
C July
D August
E September

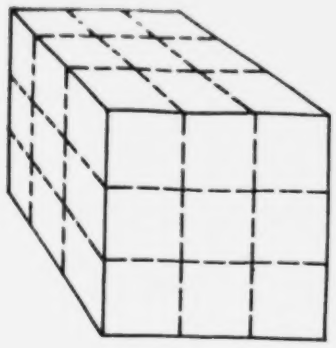
9 In a sale, all model aeroplane kits are reduced by 10%. The two kits Peter wants are usually $1.35 and 65 cents.
How much will Peter save if he buys these two kits in the sale?

A 10 cents
B 20 cents
C $1.80
D $1.90

10 Imagine that a watch is placed on top of a compass so that the '12' on the watch face is at NORTH.
Which one of the following numbers on the watch is closest to SOUTH-WEST?

A 9
B 7
C 6
D 5

11 All sides of the large cube represented in this figure are painted red. The cube is then cut along the dotted lines into 27 small cubes.



How many small cubes will **not** have any paint on them at all?

A 1
B 3
C 9
D 10

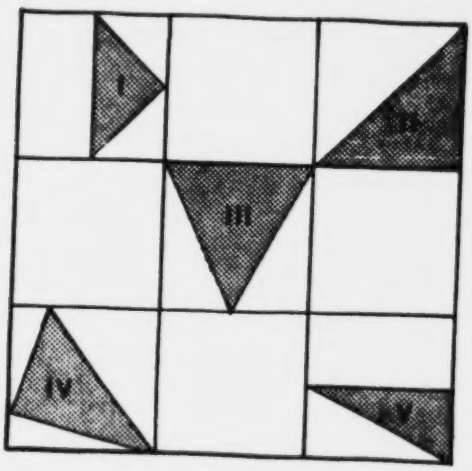
12 Consider a measurement system in which
1 jump = 5 steps
and 3 steps = 1 hop.
Three kangaroos are travelling at the following average speeds:
Hoppy — 60 hops per hour
Jumpy — 10 jumps per minute
Steppy — 2 steps per second
Which kangaroo travelled the farthest in one hour?

A Hoppy
B Jumpy
C Steppy
D Hoppy, Jumpy and Steppy travelled the same distance.

13 A cyclist took 6 seconds to cover a distance of 100 metres. Which one of the following was the cyclist's average speed?

A 10 metres per minute
B 33⅓ metres per minute
C 600 metres per minute
D 1000 metres per minute

14 In the diagram the grid squares are the same size.



Which two pairs of triangles have the same area?

A I and V, II and IV **C** II and V, III and IV
B I and IV, II and V **D** I and V, II and III

15 What is an area of 4 square metres expressed in square centimetres?

A 40 000 cm^2 **C** 400 cm^2
B 4000 cm^2 **D** 0.04 cm^2

16 $(12 \div 2) \times 2 = \triangle$

A 12 **C** 3

B 6 **D** $\frac{1}{12}$

17 $4 + 5 = (4 + 6) + (5 + \triangle)$

A 6 **C** -1

B 4 **D** -6

18 $7 - 4 = \triangle - 7$

A 11 **C** 4

B 10 **D** 3

19 $8 \div 4 = \triangle \div 8$

A 32 **C** 4

B 16 **D** 2

20 $(24 \div 6) \div 2 = \triangle \div (6 \div 2)$

A 24 **C** 6

B 12 **D** 4

21 $240 \div 15 = 480 \div \triangle$

A 30 **C** $\frac{15}{2}$

B 16 **D** $\frac{2}{15}$

22 $654 - 543 = \triangle - 654$

A 1197 **C** 543

B 765 **D** 111

23 $468 \div 234 = \triangle \div 468$

A 936 C 117

B 234 D $\frac{1}{234}$

24 $(72 \times 25) - (60 \times 25) = (72 + \triangle) \times 25$

A 60 C -12

B 12 D -60

25 $(900 \div 30) \div 10 = \triangle \div (30 \div 10)$

A 900 C 30

B 90 D 9

26 $y \times 1 = \triangle$

A $y + 1$ C 1

B y D $\frac{1}{y}$

27 $w \times 0 = \triangle$

A w C 0

B 1 D $w \div 0$

28 $(j - k) + k = \triangle$

A $j - 2k$ C $j - k$

B $j - k^2$ D j

29 $(m \div g) \times g = \triangle$

A mg C m

B $\frac{m}{g^2}$ D $\frac{m}{2g}$

30 $p + q = (p + r) + (q + \triangle)$

A s C $-r$

B r D $p + q - r$

MACQUARIE UNIVERSITY
SCHOOL OF EDUCATION

MT DRUITT LONGITUDINAL STUDY

MATHEMATICS TEST*

THERE ARE 24 QUESTIONS IN THIS MATHEMATICS TEST. ALTHOUGH SOME QUESTIONS MAY NOT BE EXACTLY THE SAME AS THE WORK YOU DO IN CLASS, YOU SHOULD TRY TO WORK OUT THE CORRECT ANSWER.

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In each question alternative answers are shown by the letters A, B, C, D or E. Mark your answer with a circle on the answer sheet.

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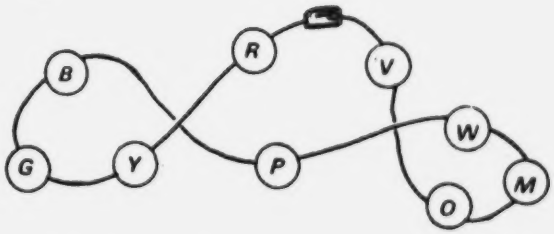
Practice Questions

P_1

Some words and numerals have the same meaning if they are read upside down. SWIMS is an example of such a word.
Which one of the following numerals means the same number, if we read it upside down?

A 8181
B 8108
C 609
D 6996

P_2



If the bead labelled V was threaded first, what was the order for threading the rest of the beads?

A W, M, O, P, Y, G, B, R
B W, M, O, P, B, G, Y, R
C O, M, W, P, Y, G, B, R
D O, M, W, P, B, G, Y, R

P_3

A person's average walking speed is 6 kilometres per hour. Which one of the following is the distance the person would travel in 1 hour?

A 2 km
B 3 km
C 6 km
D 12 km
E 24 km

P_4

Which one of the following gives 659 rounded to the nearest hundred?

A 600
B 650
C 660
D 700

Please wait until you are told to begin the test.

MATHEMATICS TEST

Practice Sheet

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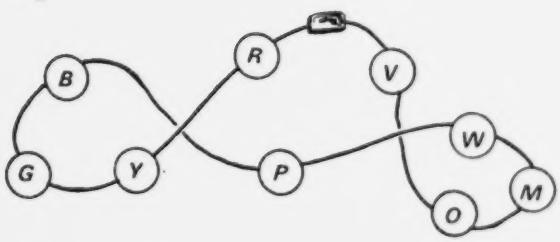
D 6996

P2



If the bead labelled V was threaded first, what was the order for threading the **rest** of the beads?

A W, M, O, P, Y, G, B, R
B W, M, O, P, B, G, Y, R
C O, M, W, P, Y, G, B, R
D O, M, W, P, B, G, Y, R

P3

A person's average walking speed is 6 kilometres per hour. Which one of the following is the distance the person would travel in 1 hour?

A 2 km
B 3 km
C 6 km
D 12 km
E 24 km

P4

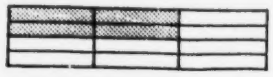
Which one of the following gives 659 rounded to the nearest hundred?

A 600
B 650
C 660
D 700

Please wait until you are told to begin the test.

1 Which one of the following diagrams does **not** have $\frac{1}{3}$ shaded?

A



B

C

D

2 A team has won 3 of the 6 games already played. If the team wins the next four games, what percentage of the games played will it then have won?

A 40%
B 50%
C 70%
D 75%

3 This number track is followed from left to right. Each circle represents the number resulting from the operation shown on the previous section of track.

The number that replaces Y would be

A 48.
B 36.
C 23.
D 19.

4 For every 3 bottles collected, a girl receives 7 cents. If she collected 210 bottles, which expression would give the total amount of money she received?

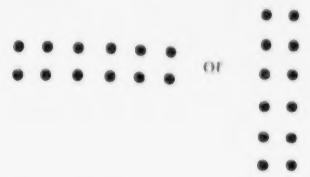
A $\frac{210 \times 3}{7}$ cents
B $\frac{3 \times 7}{210}$ cents
C $\frac{210 \times 7}{3}$ cents
D $\frac{210}{3 \times 7}$ cents

5 A school has one teacher for every 22 pupils. If the school has 88 teachers, which one of the following is the best estimate of the number of pupils attending the school?

A 352
B 700
C 1500
D 1900

6 'Rectangular' patterns can be formed from different numbers of dots if each pattern is rectangular in shape and one side has at least 2 dots.

For example, 12 dots can only form two different patterns as shown.

pattern 1  or

pattern 2 or

Which one of the following number of dots will form only **one** rectangular pattern?

A 16
B 18
C 20
D 27
E 28

7 The temperature at 8 a.m. was 7.8 °C and by 2 p.m. it had risen by 16.6 C degrees.
Which one of the following was the temperature at 2 p.m.?

A 8.8 °C
B 16.6 °C
C 23.14 °C
D 24.4 °C

8 A girl sent one card each to 21 friends. Each card cost 25 cents, and the postage for each card was 18 cents.
Which one of the following gives the total cost?

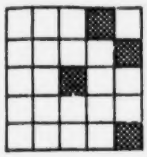
A (21 + 25) + 18 cents
B 21 × (25 + 18) cents
C 21 + (25 × 18) cents
D (21 + 25) × (21 + 18) cents

9 The diagram shows part of a number line.

Which one of the following represents the point Y?

A 1.2
B 0.4
C 0.2
D 0.04

10 The diagram shows a square divided into equal regions.



What **percentage** of the square is shaded?

A 4%
B 8%
C 16%
D 25%

11

In a class of 20 students, 35 per cent are boys and the remainder are girls. How many girls are there in the class?

A 6
B 7
C 13
D 14
E 15

12 The diagram shows part of a number line.

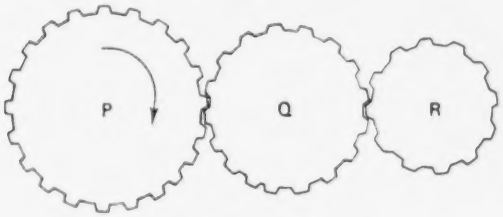
Which one of the following represents a point in the shaded part of this number line?

A $\frac{1}{2}$
B $\frac{3}{4}$
C $\frac{4}{5}$
D $\frac{1}{8}$

13 How many triangles are there in this figure?

A 3
B 4
C 5
D 6

14 P, Q, and R represent three cogwheels and P is turning in a clockwise direction.



Which one of the following is true?
A Q turns clockwise
B R turns clockwise
C Q and R turn clockwise
D Q and R turn anticlockwise

15 This clown's head was drawn on the outside of a balloon.

Which one of the following shows how the head would appear if it could be viewed from inside the balloon?

A

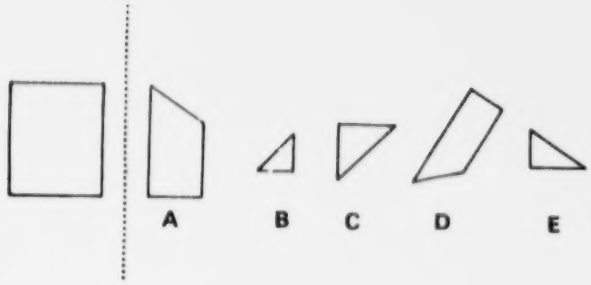
C

B

D

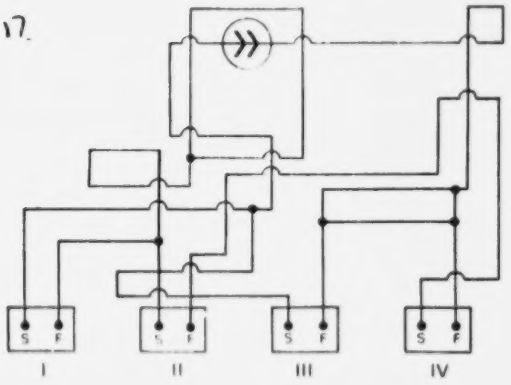
16 The shape to the left of the dotted line can be made by fitting together four of the five shapes without overlap.

Which one is **not** needed?



hen travelling along the paths of this network, changes of tracks are possible only at intersections marked with a dot (•).

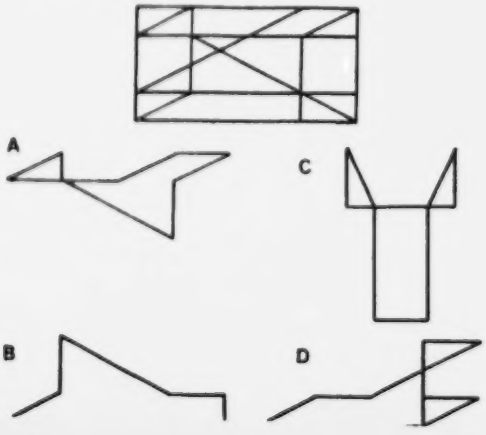
17.



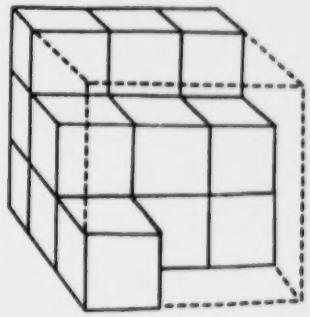
From which one of these boxes can you start at S, follow some path which passes through the circle and return to F in the same box?

A box I
B box II
C box III
D box IV

18 Which one of the following figures does **not** appear in this pattern?

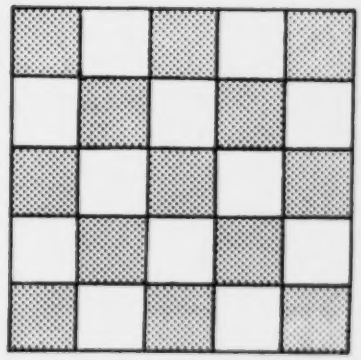


19 How many more small cubes would be needed to complete the larger cube, as outlined by the dotted lines?



A 8
B 9
C 11
D 14

20 Four students were asked to work out what fraction of this square figure is shaded.



Which one of their statements is correct?

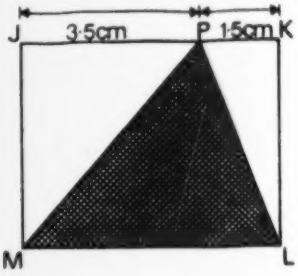
A 'Exactly half is shaded.'
B 'Just over a half is shaded.'
C 'Just under a half is shaded.'
D 'I don't think that you can work out an exact fraction.'

21

Margaret stands on a hilltop facing due north, then turns in a clockwise direction until she faces south-west. Through what angle has she turned?

A 135°
B 180°
C 225°
D 270°

22 The area of the rectangle JKLM is 20 square centimetres.



What fraction of the rectangle is the shaded area?

A $\frac{7}{10}$

B $\frac{1}{2}$

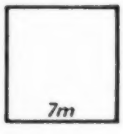
C $\frac{3}{7}$

D It cannot be calculated without more information.

23 In each of these squares, one of the dimensions has been marked to represent 7 metres. Which one of the squares would have the greatest perimeter?

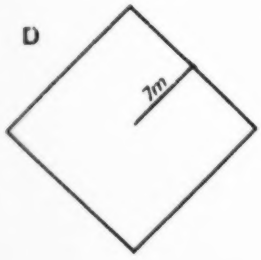
A

C



B

D



24 A square piece of paper was folded, as shown in Figures I, II, and III. Figure IV shows where a small hole was then punched through the paper.

If the paper was completely unfolded, which one of the following shows how it would look?